TECHNICAL NOTE

Scale-size calculations, energy dissipation rates and height coverage of a 404 MHz wind profiler at IMD, Pune

R. R. Joshi, Narendra Singh, S. H. Damle and G. B. Pant

The 404 MHz wind profiler at the India Meteorological Department, Pune, is the first wind profiler of this frequency in India. Therefore, it is necessary to achieve maximum possible height to get valuable data. UHF wind profilers have better height and time resolution, which enables us to make reliable and correct estimations of the wind at various altitudes at regular spacing above the observational site. The height coverage of the profiler depends upon various factors like power aperture product and the operating wavelength vis-a-vis scale sizes available in the atmosphere. In this study, daily averaged second moment data archived during clear-air conditions have been used to estimate the scale-size parameters in terms of eddy dissipation rates, which explicitly indicates that more height coverage of a profiler depends only on the availability of the scale sizes in the atmosphere to which the radar is sensitive, and that in turn depends on where in the inertial subrange the operating wavelength resides. Power aperture product at various C_n^2 values has been estimated. This note explicitly explains why the height coverage of the wind profiler at Pune is limited to 6–8 km under clear-air conditions.

A clear-air Doppler radar profiler measures the radial velocity spectrum of refractive index irregularity scatterers moving with the average wind in the radar measurement volume. The first moment of the radar velocity spectra in three orthogonal directions, one zenith and two off zenith (east and north) then leads us to estimates of zonal, meridional and vertical wind velocities and wind shear profiles. The second moment of the power spectra contains information about the turbulent velocity fluctuations. Several studies have discussed the theory and practice of using the spectral width for the estimation of turbulent intensity (energy dissipation rate, ε)[1–6]. Satheesan and Krishna Murthy[7] proceeded with a suitably long time series of fast-sampled vertical velocity values obtained by MST radar, to identify the Brunt Vaisala frequency (N) in the Fourier transform of this series. In this method, the variance σ_w^2 of the vertical velocity field is obtained by integrating the Fourier spectra of vertical velocity time series from N to the Nyquist frequency, which is presumed to cover the full inertial subrange of turbulence. Once σ_w^2 is estimated, ε is found using the relationship[8] between σ_w^2 and ε. The method works well in low wind speed (WS) conditions (WS < 10 ms^{-1}), whenever the Brunt Vaisala frequency can be unambiguously identified. The latter is a rather tricky and subjective affair, and automation of the computation is rather difficult. The method proposed by Hocking[3,9], uses the 0th moment of the velocity spectra. The method requires a reasonably well-calibrated profiler radar system and it also needs auxiliary measurements of humidity and temperature gradients obtained usually by the use of radiosonde balloon techniques. In the absence of such auxiliary data, the spectral width method for estimation of ε is normally preferred.

The observed spectral width of the signal velocity field, in a profiler, however, is also contaminated by certain non-turbulent effects. The most important non-turbulent contributors are finite beam width of the antenna and wind shear. Nastrom and Eaton[10] have evaluated these effects and the observed spectral width can be corrected appropriately to obtain only the velocity spread because of turbulence.

Definition of the problem: basic concept in turbulence

The energy dissipation rates per unit mass or the eddy dissipation rate (ε) is an integral parameter of the turbulence theory. Since ε represents the conversion of turbulent kinetic energy ultimately into heat, it is important for a broad understanding of energy flow in the atmosphere. The dissipation rate ε along with the kinematic viscosity (ν) of the atmospheric fluid determines the motion of the smallest turbulent eddies. In the entire process of cascading of the eddies from large spatial scale to smaller scales, big whirls to smaller whirls in Richardson's verse, there is a range of scales where no sources and sinks of kinetic energy exist and the energy is only transferred from larger-scale eddies to smaller-scale eddies; this particular region of scales is referred to as the inertial subrange. Therefore, while the dissipation rate ε can be measured from the energy flow through any scale within the inertial subrange, the magnitude of ε along with knowledge of the kinematic viscosity (ν) can be used to estimate the smallest scale of turbulence, the so-called Kolmogorov scale. The basic assumptions and hypothesis on turbulence theory are as follows.

(a) Kolmogorov's first similarity hypothesis on turbulence states that in every turbulent flow at sufficiently high Reynolds number, the statistics of small-scale motions has a universal form that is uniquely determined by the dissipation rate (ε) and kinematic viscosity. Based on this hypothesis and dimensional analysis approach, Kolmogorov derived the smallest eddy size/scale size of turbulence $\xi \propto (\nu^3/\varepsilon)^{1/4}$. The proportionality constant is the Kolmogorov constant = 1.5. However, Pao[11] showed that it is more appropriate to define the smallest typical length scale associated with the dissipation process as:

$$\xi_{\mathrm{P}} = \left(\frac{3k_0}{2}\right)^{3/4} \cdot \xi, \qquad (1)$$

where $k_0 = 1.5$; thus $\xi_{\mathrm{P}} \cong 2\xi$.

Table 1. Variation of kinematic viscosity with altitude

Height (km)	*Kinematic viscosity	Height (km)	*Kinematic viscosity	Height (km)	*Kinematic viscosity	Height (km)	*Kinematic viscosity
1.05	1.5890	3.45	1.9432	5.85	2.4043	8.25	2.9704
1.35	1.6286	3.75	1.9943	6.15	2.4713	8.55	3.0583
1.65	1.6693	4.05	2.0472	6.45	2.5058	8.85	3.1494
1.95	1.7114	4.35	2.1018	6.75	2.5764	9.15	3.2440
2.25	1.7549	4.65	2.1583	7.05	2.6496	9.45	3.3422
2.55	1.7997	4.95	2.2167	7.35	2.7255	9.75	3.4444
2.85	1.8460	5.25	2.2770	7.65	2.8042	10.05	3.5504
3.15	1.8939	5.55	2.3396	7.95	2.8858	10.35	3.6608

*The values of kinematic viscosity have to be multiplied by a factor of 10^{-5}.

Table 2. Experimental specifications and data-processing parameters of the wind profiler location at Pune (18.5°E, 73.85°N)

Parameter	Specifications
Co–Co antenna (phased array)	14 m × 14 m
Frequency (f)	404.37 MHz
Radar wavelength (λ)	74 cm or 0.74 m
Transmitted peak power (Pt)	16 kilowatt
Effective aperture (Ae)	80 m^2
3 dB beam width	≤ 5°
No. of beams	3 (NS, EW and zenith)
Off zenith angle or elevation (χ)	16.3° or 73.7°
Receiver path loss (α_r)	2.2 dB
Transmitter path loss (α_t)	0.8 dB
Total system temperature (T_{Op})	800 K
Maximum duty ratio (LH/HH)	3.3%/10%
Pulse width (τ) (LH/HH)	2 μs (uncoded)/16 μs (8-bit-coded of 2 μs baud length)
Inter pulse period (T) LH/HH	60 μs/160 μs
Range resolution (ΔR)	300 m
Coherent integrations (Nc)	76 (selectable)
No. of FFT points	512 (selectable)
Average incoherent integrations (ni)	10
Power aperture product (including losses)	~2×10^4 in lower mode ~7×10^4 in higher mode
Lower mode range bins (km) R	1.05–4.35
Higher mode range bins (km) R	3.15–10.35 (selectable) 3.15–4.35 range bins are common to both modes
Beam dwell time (s) (T_0) LH and HH	32.3 and 85.5

In this study, ξ_P is used as the lowest scale and is loosely defined as the Kolmogorov scale, where all the turbulent kinetic energy (TKE) is finally converted into heat by viscous damping.

The Kolmogorov theory of mechanical turbulence in homogeneous isotropic fluid is an asymptotic theory which has been shown to work well in the limit of high Reynold numbers. Reynolds number is defined as $R_{eL} \sim (e_k^2/\varepsilon\nu)$, where

$$e_k^2 = \left\{\frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})\right\}^2$$

= (Fluctuating turbulent kinetic energy)^2,

ε is the energy dissipation rate and ν the kinematic viscosity. u', v', w' are the fluctuations in the corresponding components of the wind.

(b) Under the assumption of isotropic homogeneous turbulence, one obtains from the Kolmogorov theory[11,12,15] the contribution to turbulent kinetic energy $E(k)$ at a wave number k (= $2\pi/l$, where l is the scale size of turbulence) as

$$E(k) = k_0 \varepsilon^{2/3} k^{-5/3} f_n, \qquad (2)$$

where $f_n = \exp\{-2.25(k\xi)^{4/3}\}$ and f_n accounts for the dissipation occurring as a function of $(k \cdot \xi)$. Since the India Meteorological Department (IMD), Pune wind profiler operates at 0.74 m, it is sensitive mainly to scale sizes of 0.37 m and hence for our observation $k = (2\pi/0.37)$. The direct numerical simulation shows that f_n is a close approximation.

Estimates of ε as obtained from the spectral width measurements described earlier, allow us to calculate the Kolmogorov scale length (ξ) and hence the term $k \cdot \xi$. The values of $(k \cdot \xi)$ can then be calculated as a function of height in the atmosphere. It is customary in the theory of turbulence to normalize $E(k)$ by dividing it by the factor $\xi \cdot V_\xi^2$, where

$$V_\xi = (\varepsilon \cdot \nu)^{1/4}.$$

The kinematic viscosity ν is a function of height in the atmosphere. The values of ν as a function of atmospheric height for a standard atmosphere are readily available in the literature. The values of ν as a function of height are given in Table 1 and have been used in this study to estimate ξ and normalized $E(k)$.

The wind profiler system at IMD, Pune

The wind profiler system of IMD, Pune, operates at a wavelength of 74 cm (frequency 404 MHz) and routinely provides data on hourly averaged vector winds (the first moment of the spectra) based on a consensus average of ten sets of values of zonal, meridional and vertical velocities obtained in an hour. A summary of the technical specifications and data-processing parameters of the profiler used for this study is given in Table 2. More details, including signal processing

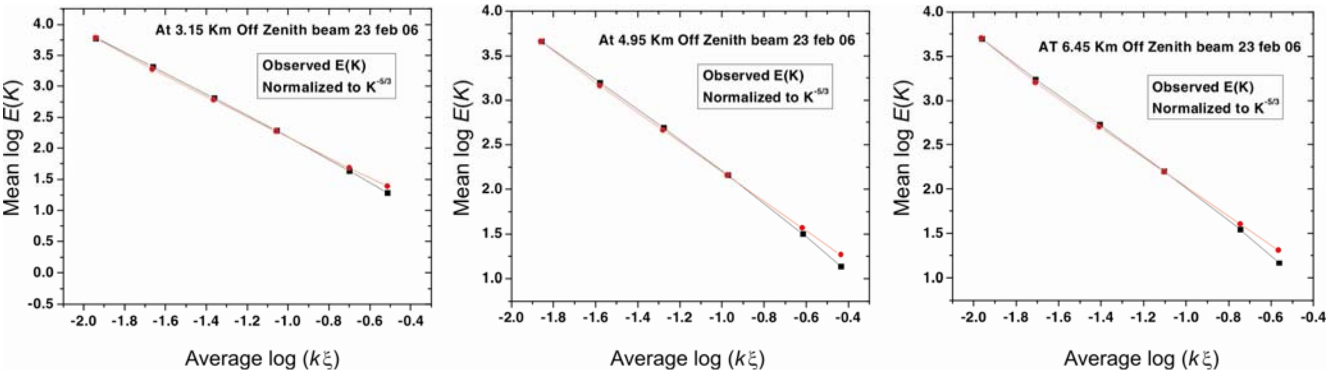

Figure 1. Deviation of $E(k)$ from its normalized values vs calculated $k \cdot \xi$ values for the 404 MHz profiler during 23 February 2006.

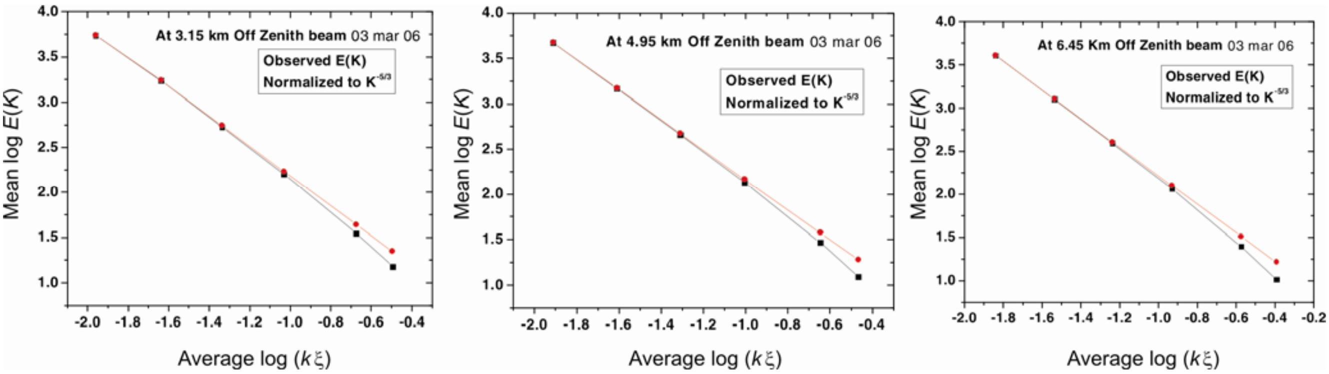

Figure 2. Same as Figure 1, but for March 2006.

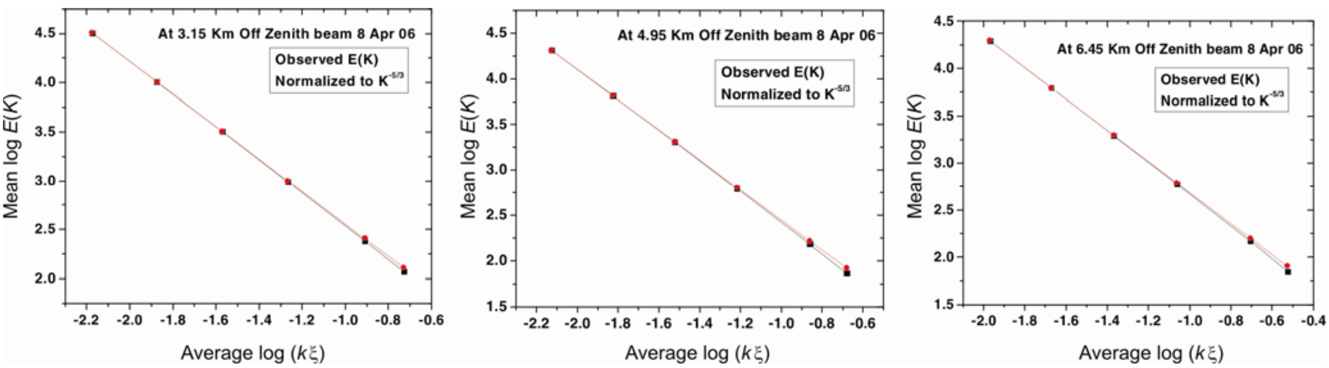

Figure 3. Same as Figure 1, but for 8 April 2006.

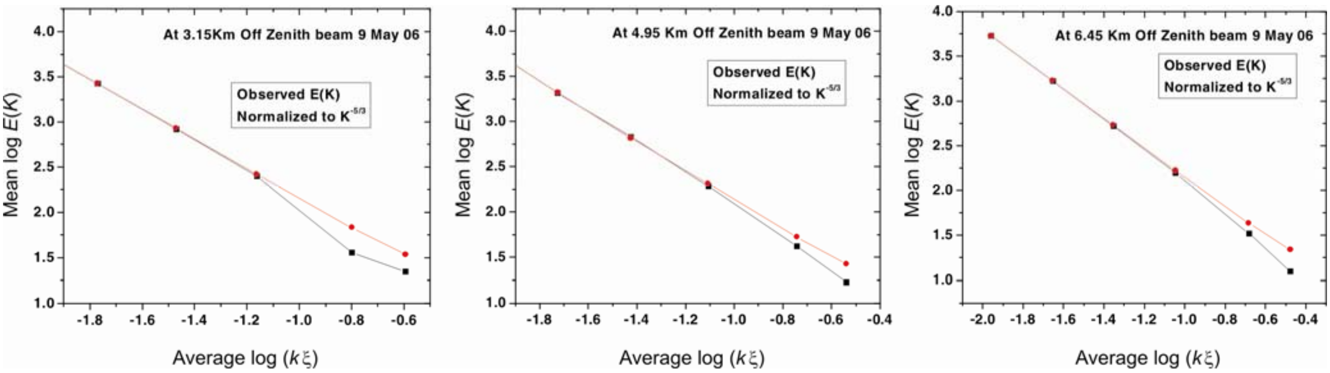

Figure 4. Same as Figure 1, but for 9 May 2006.

and quality control procedures adopted have been described by Pant *et al.*[13].

Estimates on the spectral width of the signal for all three radial beams–zenith and two off vertical (east and north) obtained every 6 min are also available as a standard processed output of the system. Each cycle of the 6 min spectra of the radial beams is the resultant of ten such spectra, averaged to improve the signal-to-noise ratio. Ten such cycles are taken for 1 h of observations. Eight observations were taken in a day during monsoon season and four during other seasons.

Data and methodology

The experimentally observed radial beam spectra were first manually edited to eliminate those spectra, particularly in the lowest two or three range bins which appear contaminated by interference and then the energy dissipation rates were calculated by the spectral width method using due corrections for non-turbulent effects[10]. Hourly averaged clear-air datasets of the Pune wind profiler for all available observations (maximum of four observations a day) on 23 February, 3 March, 8 April and 9 May 2006 have been utilized in this study. Indian MST radar data from Gadanki, Tirupati on 22 February and 4 March 2006 have also been used for estimation of $k\xi$, over the Gadanki site for comparison.

In this analysis, the daily average of ε values from the second moment estimates of off-zenith beam spectra have been used to estimate ξ and $E(k)$ values for different heights in the atmosphere. Since ξ is independent of the observing wavelength, the values of $E(k)$, at a given height (and specified location on which ξ is measured) are calculated by simply using different k values ($= 2\pi/\lambda/2 = 4\pi/\lambda$) with λ values that have been used in different profilers such as $\lambda = 5.9$ m (53 MHz), 3 m (100 MHz), 1.5 m (200 MHz), 0.74 m (404 MHz), etc. The off-zenith angle of the Pune profiler observations is 16.3°; the profiles depicting $E(k)$ at different $k \cdot \xi$ values obtained in this fashion for a given height (a specified ξ value) have been obtained for the off-zenith beams, using eqs (1) and (2). Only the off-zenith beam positions data are used because the turbulence is expected to be isotropic at the off-zenith beam angles and also that for the zenith beam, the scattering would be contaminated by Fresnel reflections apart from turbulence. Hourly averaged normalized $E(k)$ values (normalized to $\xi \cdot V_\xi^2$) versus $k \cdot \xi$ for three different heights, i.e. 3.15, 4.95 and 6.45 km on 23 February, 3 March, 8 April and 9 May 2006 are plotted (for the Pune profiler location) in logarithmic scale in Figures 1–4 respectively, for off-zenith beams. In these figures, assuming the normalized $E(k)$ values for $\lambda = 5.9$ m (53 MHz) as falling within the inertial subrange, other values normalized to this are plotted following the $k^{-5/3}$ law. The point of deviation of the $E(k)$ profile from the $k^{-5/3}$ law indicates the lower limit of the inertial subrange. The inertial subrange lower limit point is typically found at $k \cdot \xi \sim 0.1$. This means that if a profiler operates at a wavelength λ, then for the profiling operation to be within inertial subrange of turbulence it is necessary to satisfy the condition that

$$\frac{2\pi}{\lambda/2}\xi \le 0.1\text{, i.e. } \lambda \ge 40\pi\xi.$$

This would ensure that the scale size to which the radar is sensitive is larger or

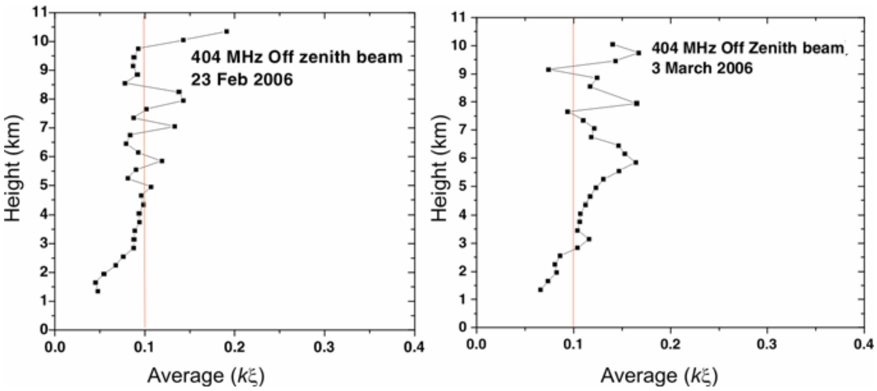

Figure 5. $k \cdot \xi$ as a function of height for 23 February and 3 March 2006.

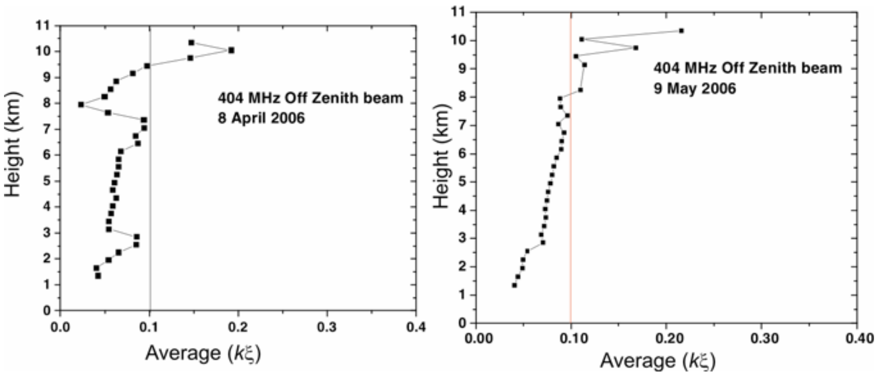

Figure 6. Same as Figure 5, but for 8 April and 9 May 2006.

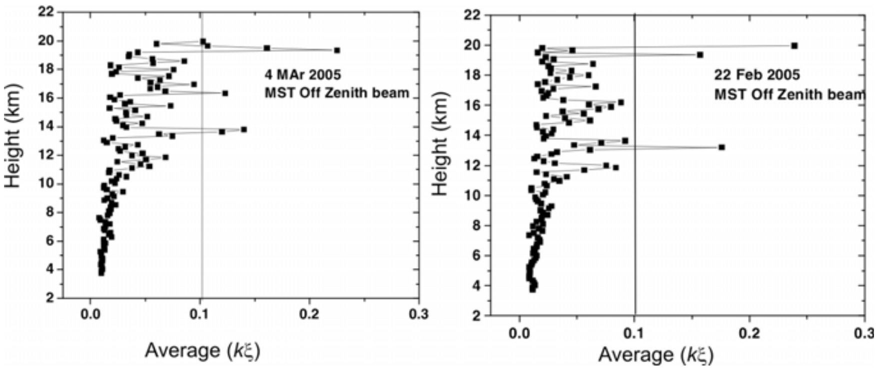

Figure 7. Height profile of average $k \cdot \xi$ for Indian MST radar for off-zenith beam.

near the lower limit of the inertial subrange of turbulence scales. Figures 5 and 6 show the height profile of $(k \cdot \xi)$ $(= 2\pi/0.37)$ for the 404 MHz profiler at Pune based on observed one-day average values of $k \cdot \xi$ at different heights for off-zenith beams. It clearly shows that beyond a height of about 8–10 m, the assumption of inertial subrange $(k \cdot \xi \leq 0.1)$ is no longer valid for this profiler probing the atmosphere over the Pune site. In order to verify that the $k \cdot \xi$ values for a 53 MHz profiler are indeed much less than 0.1, data from the MST radar operating at 53 MHz at NARL, Gadanki, near Tirupati were examined/analysed to obtain average ε, $E(k)$ and ξ values over Tirupati for height up to 19/20 m. Figure 7 shows the height profiles of $(k \cdot \xi)$ for off-zenith beams on 22 February and 4 March 2005 for MST radar. It can be seen that $k \cdot \xi << 0.1$, indicating that the MST radar at 53 MHz does operate well within the inertial subrange of turbulence over the Gadanki site.

Power aperture product at various C_n^2 values

To get the desired height coverage for specified C_n^2 values, the desired power aperture product (PAP) in terms of known radar parameters was estimated. The detectability S/N ratio for the 404 MHz wind profiler at Pune was obtained from the formulation of Gossard and Strauch[14]. C_n^2 was calculated using the radar parameters and known constants (Table 2) for the Pune profiler as:

$$C_n^2 = m\frac{2K_{\mathrm{B}}T_{Op}R^2 \cdot \sigma_V^2(SNR)_{dt}}{0.38K(\alpha_{\mathrm{r}}\alpha_{\mathrm{t}}\overline{P}_{\mathrm{t}}A_{\mathrm{e}})\lambda^{1/6}\Delta R}\sqrt{\frac{\pi}{T_0\Delta V}}, \tag{3}$$

where K_{B} is the Boltzman constant; σ_V the SD of the signal spectrum; ΔV the spectral measurement resolution (m/s); K a constant which depends on the radar beam width, assuming a Gaussian beam = 0.0354 (Gossard and Strauch[14]); T_0 the beam dwell time and m the number of bauds in a coded pulse; for uncoded pulse, $m = 1$.

Using eq. (3), the average PAP for various radar frequencies in operation, for specified C_n^2 values for Pune profiler was calculated as:

$$PAP = (\overline{P}_{\mathrm{t}}A_{\mathrm{e}}\alpha_{\mathrm{r}}\alpha_{\mathrm{t}})$$
$$= 5.5552 \times 10^{-21} \times \frac{R_{\max}^2}{C_n^2},$$

where $R_{\max}$ is the maximum height coverage achieved $(\mathrm{S/N})_{\mathrm{dect}} = 1$.

Figure 8 shows the actual PAP for the Pune wind profiler at $C_n^2 = 10^{-17}$ $(\mathrm{m}^{-2/3})$. These estimates are theoretically ideal as long as the radar wavelength satisfies the inertial subrange condition mentioned earlier. When $k \cdot \xi$ is more than 0.1, the dissipation loss increases and correspondingly the effective C_n^2 is less than 10^{-17}, and the height coverage tends to reduce.

Results and discussion

In our analysis we have used the daily average of ε values from the second moment estimates of off-zenith beam

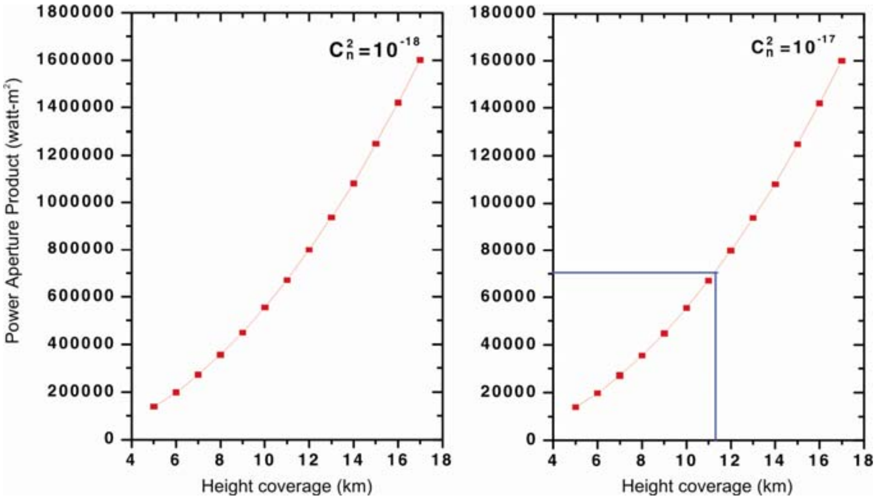

Figure 8. Power aperture product and height coverage.

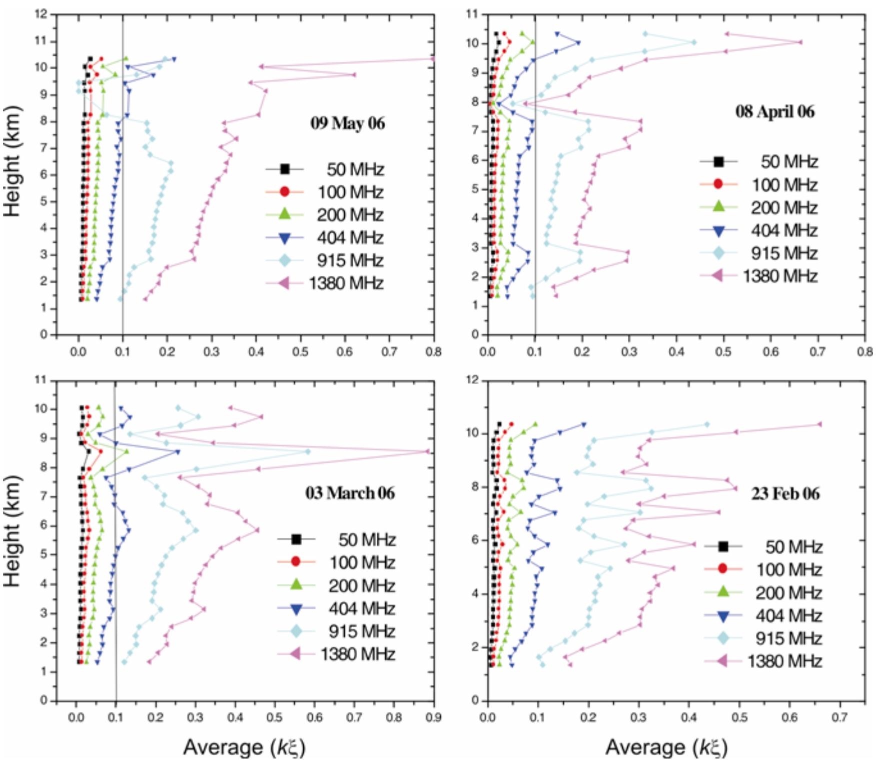

Figure 9. Daily average $(k \cdot \xi)$ as a function of height for off-zenith beam at various operating frequencies for 9 May, 8 April, 3 March and 23 February 2006.

spectra of the 404 MHz profiler at Pune and estimated ξ and $E(k)$ values for different heights in the atmosphere over the Pune site. Since ξ is independent of the observing wavelength, the values of $E(k)$ at a given height are calculated as a function of k, by simply using different k ($= 2\pi/\lambda/2$) with λ values that have been used in different profilers such as $\lambda = 5.9$ m (53 MHz), 3 m (100 MHz), 1.5 m (200 MHz), 0.74 m (404 MHz), etc. We thus have profiles depicting $E(k)$ at different $k \cdot \xi$ values for a given height (a specified ξ value). As is the practice in turbulence theory, the TKE values are normalized by dividing by a factor ($\xi \cdot V_\xi^2$).

Figure 9 for off-zenith beams gives the complete projection of estimated $k \cdot \xi$ versus height, using the actual measured values of ε at various heights over Pune. These results indicate that the typical height coverage for Pune profiler under clear-air conditions is between 8 and 10 km for $C_n^2 = 10^{-17}$; in practice, the profiler appears to achieve this over the Pune site. If the operating wavelength is well within the inertial subrange, one would have expected a coverage closer to 12 km at $C_n^2 = 10^{-17}$.

Conclusion

Estimation of ε from velocity variance (σ_w^2) observations using a profiler has been shown to be useful in estimating the lower bound of inertial subrange of turbulence in the atmosphere and thus provides a valuable input for selection of operating frequency of a profiler for a given site. The explicit relationship between ε and σ_w^2 as derived by Frisch and Clifford[2], and Gossard and Strauch[4], and as used by us, tacitly assumes that the outer scale size of turbulence (L_O) is much larger than the maximum dimension of the radar beam sampling volume (L_R) and that the radar wavelength lies well within the inertial subrange of the turbulence scales of motion. In the present study we have brought out the effect on height coverage of a profiler operating at a wavelength λ at a site, when the Kolmogorov scale length over the site is less than $\lambda/2$. When $L_O < L_R$, the observed spectral width in the velocity field still represents turbulence variance, but to relate it to the eddy dissipation rate one needs to use suitable model expressions for TKE spectrum, inclusive of the production range of turbulence which is a function of kL_O, along with the established inertial subrange spectrum with $k^{-5/3}$ dependence, k being the wave number. Several studies[6,9] have pointed out that when the finite beam width and wind shear corrections are applied to the observed velocity variance, the resultant variance turns out to be negative. In such a case, the Frisch and Clifford[2] formulation cannot be utilized to estimate ε. One then assumes that the correction formulation overestimates the required correction, leading to the above dilemma. It is conjectured that the cases of so-called overestimation of the correction for non-turbulent effects may occur when $L_O << L_R$. An explicit procedure for estimation of L_O independently and then relating it to ε and σ^2 has, to our knowledge, is not available so far and needs to be addressed in future.

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